

**Hong Kong Mathematics Olympiad (2015/2016)**  
**Final Event 1 (Individual)**

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**FOR OFFICIAL USE**

Score for accuracy	<input type="text"/>	×	Mult. factor for speed	<input type="text"/>	=	<input type="text"/>	Team No.	<input type="text"/>
			+	Bonus score		<input type="text"/>	Time	<input type="text"/>
							Min.	Sec.
						<input type="text"/>		

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.  
除非特別聲明，答案須用數字表達，並化至最簡。

1. 解方程  $\log_5 a + \log_3 a = \log_5 a \cdot \log_3 a$ ，其中  $a > 1$  為實數。

Solve the equation  $\log_5 a + \log_3 a = \log_5 a \cdot \log_3 a$  for real number  $a > 1$ .

$a =$

2. 若  $\sqrt{b} = \sqrt{8 + \sqrt{a}} + \sqrt{8 - \sqrt{a}}$ ，求  $b$  的實數值。

If  $\sqrt{b} = \sqrt{8 + \sqrt{a}} + \sqrt{8 - \sqrt{a}}$ , determine the real value of  $b$ .

$b =$

3. 若方程  $x^2 - cx + b = 0$  有兩實數根及兩根之差為 1，求兩根之和的最大可能值  $c$ 。

If the equation  $x^2 - cx + b = 0$  has two distinct real roots and their difference is 1, determine the greatest possible value of the sum of the roots,  $c$ .

$c =$

4. 設  $d = \overline{xyz}$  為一不能被 10 整除的三位數。若  $\overline{xyz}$  與  $\overline{zyx}$  之和可被  $c$  整除，求此整數的最大可能值  $d$ 。

Let  $d = \overline{xyz}$  be a three-digit integer that is **not** divisible by 10. If the sum of the integers  $\overline{xyz}$  and  $\overline{zyx}$  is divisible by  $c$ , determine the greatest possible value of such an integer  $d$ .

$d =$

**Hong Kong Mathematics Olympiad (2015/2016)**  
**Final Event 2 (Individual)**

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1. 設一個等邊三角形及一個正六邊形的周界比率為 1:1。若三角形與六邊形的面積比率為  $2:a$ ，求  $a$  的值。

Let the ratio of the perimeter of an equilateral triangle to the perimeter of a regular hexagon be 1:1. If the ratio of the area of the triangle to the area of the hexagon is  $2:a$ , determine the value of  $a$ .

$a =$

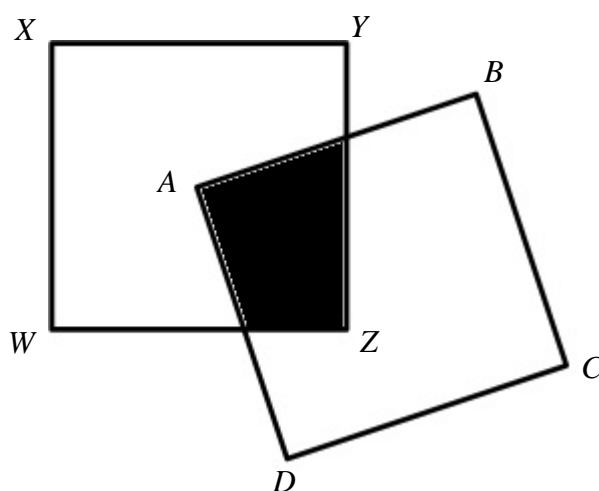
2. 求  $b = \left(\log_2(a^2) + \log_4\left(\frac{1}{a^2}\right)\right) \times \left(\log_a 2 + \log_{a^2}\left(\frac{1}{2}\right)\right)$  的值。

Determine the value of  $b = \left(\log_2(a^2) + \log_4\left(\frac{1}{a^2}\right)\right) \times \left(\log_a 2 + \log_{a^2}\left(\frac{1}{2}\right)\right)$ .

$b =$

3. 在下圖中，正方形  $ABCD$  及  $XYZW$  相等而且互相交疊使得頂點  $A$  位在  $XYZW$  的中心及線段  $AB$  將線段  $YZ$  邊分為  $1:2$ 。若  $XYZW$  的面積與交疊部分的面積比率為  $c:1$ ，求  $c$  的值。

In the figure below, identical squares  $ABCD$  and  $XYZW$  overlap each other in such a way that the vertex  $A$  is at the centre of  $XYZW$  and the line segment  $AB$  cuts line segment  $YZ$  into  $1:2$ . If the ratio of the area of  $XYZW$  to the area of the overlapped region is  $c:1$ , determine the value of  $c$ .



$c =$

4. 若  $76$  與  $d$  的最小公倍數 (L.C.M.) 為  $456$  及  $76$  與  $d$  的最大公因數 (H.C.F.) 為  $c$  求正整數  $d$  的值。

If the least common multiple (L.C.M.) of  $76$  and  $d$  is  $456$  and the highest common factor (H.C.F.) of  $76$  and  $d$  is  $c$ , determine the value of the positive integer  $d$ .

$d =$

**Hong Kong Mathematics Olympiad (2015/2016)**  
**Final Event 3 (Individual)**

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			+	Bonus score		<div></div>	Time	<div></div>
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除非特別聲明，答案須用數字表達，並化至最簡。

1. 若  $f(x) = x^4 + x^3 + x^2 + x + 1$ ，求  $f(x^5)$  除以  $f(x)$  的餘值  $a$ 。

If  $f(x) = x^4 + x^3 + x^2 + x + 1$ , determine the remainder  $a$  of  $f(x^5)$  divided by  $f(x)$ .

$a =$

2. 設  $n$  為整數。求  $n^a - n$  除以 30 的餘值  $b$ 。

Let  $n$  be an integer. Determine the remainder  $b$  of  $n^a - n$  divided by 30.

$b =$



3. 若  $0 < x < 1$ ，求

$$c = \left( \frac{\sqrt{1+x}}{\sqrt{1+x} - \sqrt{1-x}} + \frac{1-x}{\sqrt{1-x^2} + x - 1} \right) \times \left( \sqrt{\frac{1}{x^2 - b^2} - 1} - \frac{1}{x - b} \right)$$

的值。

If  $0 < x < 1$ , determine the value of

$$c = \left( \frac{\sqrt{1+x}}{\sqrt{1+x} - \sqrt{1-x}} + \frac{1-x}{\sqrt{1-x^2} + x - 1} \right) \times \left( \sqrt{\frac{1}{x^2 - b^2} - 1} - \frac{1}{x - b} \right).$$

$c =$

4. 若實數  $x$  及  $y$  滿足方程  $2 \log_{10}(x + 2cy) = \log_{10} x + \log_{10} y$ ，

求  $d = \frac{x}{y}$  的值。

If real numbers  $x$  and  $y$  satisfy the equation  $2 \log_{10}(x + 2cy) = \log_{10} x + \log_{10} y$ ,

determine the value of  $d = \frac{x}{y}$ .

$d =$

**Hong Kong Mathematics Olympiad (2015/2016)**  
**Final Event 4 (Individual)**

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			Total score			<input type="text"/>	Min.	Sec.

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.  
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若  $m$  和  $n$  為正整數及  $a = \log_2 \left( \left( \frac{m^4 n^{-4}}{m^{-1} n} \right)^{-3} \div \left( \frac{m^{-2} n^2}{m n^{-1}} \right)^5 \right)$ , 求  $a$  的值。

If  $m$  and  $n$  are positive integers and  $a = \log_2 \left( \left( \frac{m^4 n^{-4}}{m^{-1} n} \right)^{-3} \div \left( \frac{m^{-2} n^2}{m n^{-1}} \right)^5 \right)$ , determine the value of  $a$ .

$a =$

2. 當整數  $1108 + a$ ,  $1453$ ,  $1844 + 2a$  及  $2281$  除以正整數  $n$  都得相同餘數  $b$ , 求  $b$  的值。

When the integers  $1108 + a$ ,  $1453$ ,  $1844 + 2a$  and  $2281$  divided by some positive integer  $n$ , they all get the same remainder  $b$ . Determine the value of the remainder  $b$ .

$b =$

3. 若  $\frac{6}{b} < x < \frac{10}{b}$ , 求  $c = \sqrt{x^2 - 2x + 1} + \sqrt{x^2 - 6x + 9}$  的最大可能值。

If  $\frac{6}{b} < x < \frac{10}{b}$ , determine the greatest possible value of  $c = \sqrt{x^2 - 2x + 1} + \sqrt{x^2 - 6x + 9}$ .

$c =$

4. 求  $1 + 3^c + (3^c)^2 + (3^c)^3 + (3^c)^4$  除以  $1 + 3 + 3^2 + 3^3 + 3^4$  的餘值  $d$ 。

Determine the remainder  $d$  of  $1 + 3^c + (3^c)^2 + (3^c)^3 + (3^c)^4$  divided by  $1 + 3 + 3^2 + 3^3 + 3^4$ .

$d =$